Disjunctive Programming as a tool for convexifying nonconvex sets

Plenary speaker: Egon Balas

Linear Programming, born during the last world war, was quickly generalized to convex nonlinear programming; but the lack of convexity turned out to be the stumbling block which separates tractable (i.e. polynomially solvable) problems from intractable ones (whose solution requires exponentially many steps). Nonconvex problems can be reformulated as integer programs, which belong to this intractable class. Disjunctive programming formulates the integrality conditions as disjunctions, and represents the first major inroad into dealing with nonconvexity: disjunctive sets, brought to the form of unions of polyhedra, are nonconvex sets whose convex hull has a compact representation in a space of dimension linear in the number of polyhedra in the union.The disjunctive or lift-and-project approach to cutting plane theory has provided tools that have contributed decisively to the revolution in the state of the art of integer programming that took place starting with the 1990's. In the case of lift-and-project (L&P) cuts from split disjunctions, a correspondence established between bases of the higher dimensional cut generating linear program and those of the original LP relaxation have made it possible to generate L&P cuts directly from the original LP tableau, without recourse to the higher-dimensional representation. This has led to efficient implementations of L&P cuts in public (COIN_OR) and commercial (XPRESS, CPLEX, MOPTA) MIP solvers.

Recent research, focused on establishing similar results for L&P cuts from more general disjunctions, has led to rather different conclusions. Namely, while easily verifiable conditions were established for a L&P cut from a general disjunction D to be regular, i.e. equivalent to a standard intersection cut from a polyhedral counterpart of D, in the absence of those conditions the L&P cut, in this case termed irregular, has remarkable properties (like cutting part of the corner polyhedron). Furthermore, far from being exceptional, irregular cuts turn out to be more frequent than regular ones.

While irregular L&P cuts cannot be generated by pivoting in the LP tableau, they can be generated as final point cuts from general disjunctions – a recently studied class that promises to bring about an organic synthesis of branch and bound (B&Bd) with cutting plane theory. These cuts are derived from the reverse polar of the disjunctive set defined by the active branches of a B&Bd tree, which can be represented by a set of inequalities in the space of the original MIP. This procedure – more efficient than its L&P counterpart - can be used to capture information from a partial B&Bd tree in the form of cuts valid for the entire tree.